# MECHANISM INVOLVED IN FORMATION OF CONDENSATION SHOCKS

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The article discusses the mechanism underlying the formation of condensation shocks. It is shown that the formation of a condensation shock is seriously affected by the increase in the speed of sound in the zone of intense condensation.

The likelihood that some condensation process will be materialized is governed by the second law of thermodynamics, i.e., by the statement that the entropy of the working fluid increases in the passage across the shock front. This statement is satisfied by both adiabatic shocks in single-phase supersonic streams and by condensation shocks occurring in a supersonic stream of supercooled vapor [1, 2].

In the case of adiabatic shocks in a single-phase medium, however, there is available a somewhat different approach which accounts for the nature of the shock process and for the underlying mechanism. The gist of this approach is that when there exists a pressure distribution, such as indicated in Fig. 1a, in some adiabatic system, the wave of increasing pressure must inevitably become transformed into a condensation shock. Actually, the disturbances belonging to the higher-pressure zones propagate at a higher velocity (the temperature in those zones is higher) than disturbances belonging to lower-pressure zones (traveling wave). Contrariwise, if there exists a pressure distribution such as displayed in Fig. 1b, in some adiabatic system, then this distribution will be inevitably transformed in such a way that the gradient of the pressure in the disturbed region will diminish in absolute magnitude. In effect, the disturbances belonging to the lower-pressure zones will move at a slower velocity, and the extent of the disturbed zone will increase. This example is often invoked in gas dynamics in order to illustrate the point that rarefaction shocks cannot exist.

When an oblique shock wave exists in a single-phase stream, the speed of sound increases and the flowspeed of the stream slows down, so that the slope of the characteristics must become steeper with increasing pressure. This would have the result that the subsequent characteristic would move out ahead of the preceding one. But in that case the cause would be leading the consequence – the characteristics must merge into an adiabatic shock. A similar proof is invoked in the discussion of the process by which a shock wave forms in a supersonic stream flowing around a concave wall.

Consequently, in the case of adiabatic shocks occurring in a supersonic single-phase stream, in addition to the purely thermodynamic justification for changes in the stream parameters in the shock, we also have to deal with clearcut gas-dynamical concepts which go a long way toward revealing the mechanism operative in the formation of the shock, even in such a comparatively complicated instance as flow of a supersonic stream around a concave wall.

We confront an entirely different pattern in the case of condensation shocks. According to currently entertained concepts [3, 4], the reason behind the appearance of this mode of shock wave is provided by heterogeneous fluctuations resulting in the formation of nucleation droplets whose number and size increase as the stream expands, and at a particularly rapid rate starting with a certain cross section. Leaving a detailed analysis of these concepts aside for the moment, we must note for the time being that such gas-dynamical categories as the speed of sound, the Mach angle, etc., do not come into play here at all. But experimental observations have shown that condensation shocks are usually oblique shocks [5], that their

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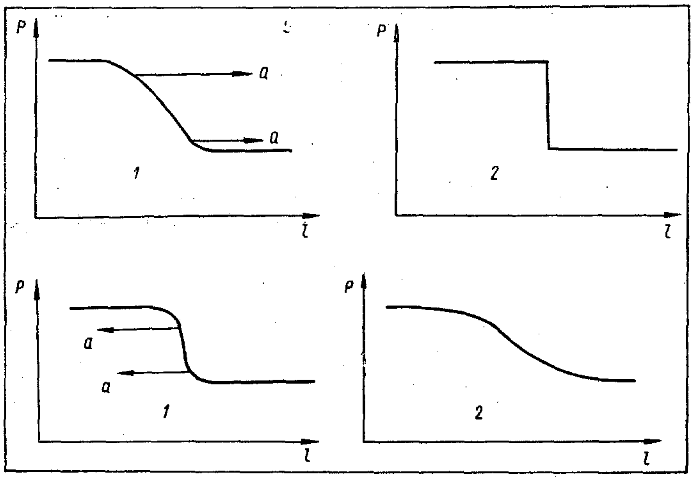


Fig. 1. Transformation of compression waves into condensation shock (a) and of rarefaction waves into the same system with a smaller pressure gradient (b): top: 1) initial pressure distribution; 2) final pressure distribution; bottom: 1) initial pressure distribution; 2) pressure distribution with lower gradient.

angle of inclination depends on a whole series of actors, and that they are clearly associated with wave processes occurring in the wake of the condensation shock.

This emphasizes the need to throw some light on the mechanism at work in the formation of a condensation shock, taking gas dynamics as our point of departure, similarly to the approach relied on in the generally familiar schemata of formation of adiabatic shocks in supersonic single-phase streams.

We are required to determine the speed of sound in a nonequilibrium supercooled vapor.

The speed of sound moving through moist vapor is given by the formula [2]

$$a^2 = \frac{1}{x\left(1+\frac{1-x}{x}\frac{c_W}{c_n}\right)}\frac{dp}{d\rho}. \tag{1}$$

To simplify the analysis, we assume that the supercooled vapor is obtained as a result of adiabatic expansion of dry saturated vapor or superheated vapor. In that case coarsely disperse moisture will be absent in the stream, and the condensation centers present in the stream will be very small in size, so that the flowspeeds of the nonsupercooled vapor and of the nuclei of the liquid phase will be demonstrably equal under those conditions, i.e., $c_n = c_W$.

We then readily find, from Eq. (1)

$$\frac{1}{a^2} = \frac{d\rho}{dp}. \tag{2}$$

With a high degree of accuracy (since the volume occupied by the liquid phase is quite restricted), the density of the supercooled vapor can be represented as

$$\rho = \rho_n + nv_d\rho_W. \tag{3}$$

Formula (3) can be restated as

$$\rho = \rho_n(1+y),$$

where y is the amount of liquid phase in the condensation nuclei per unit mass of supercooled vapor proper. We then have, from Eq. (2)

$$\frac{1}{a^2} = (1+y)\frac{d\rho_n}{dp} + \rho_n\frac{dy}{dp}. \tag{4}$$

Note that, if the process by which the disturbance propagates is adiabatic in the case of the stream of supercooled vapor, then it is accompanied by an input of heat from the condensible phase in the case of the vapor phase proper, i.e., when determining the derivative we have to take the fact that heat is supplied to the vapor medium into account.

The further discussion will be limited to the range of moderate pressures when the equation of state of an ideal gas is applicable to the vapor phase. In that case

$$\frac{d\rho_n}{dp}=\frac{1}{RT}\left(1-\frac{p}{T}\frac{dT}{dp}\right). \tag{5}$$

The derivative dT/dp is determined from the first law of thermodynamics, written in the following form:

$$rdy=c_p dT-vdp. \tag{6}$$

When we take Eqs. (5) and (6) into account, we can reduce the equation for the speed of sound to the following form:

$$\frac{1}{a^2}=\frac{1}{kRT}\left[(1+y)\left(1-\frac{p}{T}\frac{r}{c_v}\frac{dy}{dp}\right)+kp\frac{dy}{dp}\right].$$

We then end up with

$$a^2=\frac{a^2_{\mathrm{ad}}}{1-\left(\frac{r}{u}-k\right)\frac{dy}{d\ln p}}. \tag{7}$$

Here $u = c_V T$ is the internal energy. In the last equation, the fact that $y \approx 0$ in the case of a supercooled vapor is taken into account, and the equation $a^2_{\mathrm{ad}} = kRT$ represents the adiabatic speed of sound.

We now present the estimate of the correction to the adiabatic speed of sound.

The quantities appearing in Eq. (7) are estimated on the basis of the parameters of the condensation shock. According to experimental data [6], the pressure variation in the condensation shock $\Delta p$ = (0.2 to 0.3) $p_1$, while $\Delta y$ = 0.03 to 0.04. In the case of a condensation shock lying in the pressure zone ($p_1$) from 0.1 to 0.5 atm (and it is precisely at those pressures that the broad experiments on condensation shocks propagating through water vapor have been staged), the ratio r/u fluctuates between 4 and 3 in the case of water.

Substituting those quantities into Eq. (7), we see readily that

$$a=(1.1-1.4)\ a_{\mathrm{ad}}$$

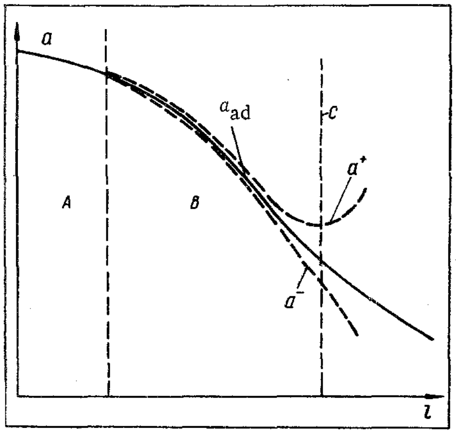


Fig. 2. Typical distribution of speeds of sound in Laval nozzle: zone A) superheated vapor; zone B) supercooled vapor; C) condensation shock; the zone $da^+/dl \approx 0$ corresponds to the site where the condensation shock originates.

The correction for the processes typical of a nonequilibrium supercooled vapor thus turns out to be quite essential in the zone of a condensation shock.

It must be emphasized that this correction can be either positive or negative, i.e., it may lead to a speed of sound either faster or slower than the adiabatic speed of sound. Actually, as a supercooled vapor flows (adiabatic expansion takes place from the state of saturated vapor or from the superheated vapor region), the moisture of the vapor can only increase because of heterogeneous fluctuations and because of the growth of the condensation nuclei formed earlier, i.e., dy > 0 in a supercooled vapor containing no coarsely disperse droplets. On the other hand, the sign of dp can be either positive (compression wave) or negative (rarefaction wave). This means that the speed at which the compression disturbance propagates ($a^+$) through a nonequilibrium supercooled vapor will be faster than the speed at which the rarefaction disturbance ($a^-$) propagates. The size of the discrepancy between the two speeds of sound is decisively influenced by the value of the derivative (dy/dp). The number of condensation nuclei present is still small immediately after the stream is intersected by the top boundary curve, and nucleation takes place at a comparatively slow rate because of the slight supercooling in progress. The adiabatic speed of sound, the speed of sound in the case of the compression wave, and the speed of sound in the case of the rarefaction wave are all practically the same in that zone. In the immediate vicinity of the condensation shock (upstream of the shock front), the absolute value of dy/dp begins to climb steeply upward; the three speeds of sound referred to naturally begin diverging substantially under those conditions.

The approximate behavior of the sound-speed curves as vapor flows through a Laval nozzle is shown in Fig. 2. The speed of sound for a compression wave must begin to increase, theoretically speaking, starting at some instant. But we know that the increase in the absolute value of dy/dp is particularly steep in the zone of the condensation shock, and accordingly a fairly rapid increase in the speed of sound for a condensation wave is found to occur in that zone. This means that a statement similar to the one considered above in the case of an adiabatic condensation shock in a single-phase stream holds in this case as well.

In other words, the portion of the flow (here we have in mind shock-free flow) corresponding to increasing $a^+$ cannot be realized under real conditions. In effect, an increase in $a^+$ means a decrease in the Mach number in the case of compression, an increase in the slope of the characteristic, and a conversion of compression waves to a condensation shock. It must be pointed out that an increase in dy/dp over the portion where intensive condensation begins (ahead of the front of the condensation shock) takes place in response to a comparatively slight variation in flowspeed and a considerable rise in the pressure. The compression waves of course merge into a condensation shock under those conditions.

The condition $da^+/dp = 0$ or $da^+/dl = 0$ thereby determines the site where the condensation shock originates. The condensation shock must inevitably appear slightly below the point where $da^+/dl = 0$. This allows us to dispense with the direct reliance on Oswatitsch integral formulas [4] in order to determine the site where the condensation shock originates. This statement constitutes the formulation of the gas-dynamic method of dealing with the mechanism behind the condensation shock, and determining the site where the condensation shock originates. The role played by the kinetics of phase transitions is not diminished in any way, since it still has a decisive effect on the value of dy/dp. This means that the prehistory of the flow determines the value of dy/dp for that cross section, in terms of the kinetics of the phase transition.

It must be pointed out that the value of dy/dp, and consequently the values of $a^+$ and $a^-$, appear to be affected to some extent by the shape of the disturbed wave. However, the problem of what effect the shape of the disturbed wave would have on the speed of sound is not decisive, since it in no way constitutes any fundamental change in the important concepts relating to the site where the condensation shock originates ($da^+/dl = 0$) or the pattern of formation of the condensation shock.

## NOTATION

$a$ is the speed of sound;
T is the temperature;
p is the pressure;
$\rho$ is the density;
x is the degree of dryness;
$v_d$ is the average volume of nucleation droplet;
n is the number of condensation nuclei per unit volume;
k is the isoentropic index;
$c_v$ is the heat capacity at constant volume;
r is the latent heat of vaporization;
R is the gas constant;
u is the internal energy;
q is the quantity of heat;
c is the speed.

Subscripts

n is the vapor phase;
w is the liquid phase.

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